Overcoming Identified Learning Difficulties in Trigonometry Using Casio Classwiz fx-991EX

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Abstract

The objective of research was to find out the trigonometry learning difficulty and the solution to the learning difficulty with classwiz fx-991EX help. The subject of research consisted of 96 10th graders of Public Vocational High School 2 Wonosari in the school year of 2017/2018. The research method employed was descriptive qualitative one. The result of research showed that learning difficulty was related to: (a) reasoning & understanding of prerequisite material (geometry and real number operation) including: symbol, distance, Pythagorean theorem, circle, triangle, angle in radian and degree, pi concept, and root form rationalization, (b) reasoning and understanding of function including: representation (numeric, graphic, symbolic/algebraic, verbal), and inter-representation relation, (c) reasoning and understanding of trigonometry function including: definition of function based on unit, definition of function based on right triangle sides ratio, multiple representation and property in trigonometry main function. Data of diagnostic test result was supported with result of interview with informants selected representing any types of error in diagnostic test. Learning difficulty revealed was improved using remedial teaching with casio classwiz fx-991EX help.

1. Introduction

This paper is a continuation of our studies of how to teach trigonometry with reasoning and *sense making*. It can be found that the learning emphasizing on reasoning and sense making is unforgettable to students and can be a capital to construct the following concept or to deal with the problem in real world [12]. The fact of learning without understanding has been a common phenomenon in mathematics classes since 1930s. The same fact occurs in mathematics learning in Indonesia [12]. The same fact occurs in mathematics learning in Indonesia. Considering the result of previous research [15], it can be found that the mean score of Indonesian students' achievement is always on the last rank group since 2000-2012. The result of research [15] measures mathematics literacy achievement in identifying, understanding, and using mathematics basics needed in daily life. In other words, Indonesian students, including those in the area where the author teaches, generally have relatively low identification, understanding, and application abilities compared with other participant states.

To solve the problem, the author explores the factors causing students' learning difficulty and look for alternative learning solution by emphasizing on reasoning and sense making. An alternative solution supporting the learning process to run according to standard process is to optimize infrastructure and learning media that can facilitate teachers and students' needs. Kultur [9] recommends some ways to deal with students and teachers' learning difficulty and to improve the meaningful learning by facilitating the conceptual learning in-depth, to connect prerequisite knowledge to new knowledge. Meanwhile another study [8] investigated some representations of function dynamically using calculator and computer software. This study found that students and teachers have access to trigonometry learning in 21st century, thereby can conduct learning in some ways using technology. Because technology use as a cognitive tool is very helpful in learning, the author discusses trigonometry learning difficulty and the solution to it using Casio Classwiz fx-991EX calculator. Thus, the students' learning difficulty is expected to be solved and the learning will be more meaningful to students.

2. Trigonometry Learning Difficulty

Mathematics learning difficulty is defined as a real form of difficulty in the term of proficiency and ability to apply mathematics to daily life [13]. Learning difficulty condition can be detected from several symptoms: academic, cognitive, physical, and social symptoms in learning [16]. There are two objects the students can acquire in learning mathematics to be applied to daily life: direct and indirect ones. Direct object of mathematics includes fact, concept, operation (skills) and principle. Thus, trigonometry learning difficulty can diagnose students' difficulty in using concept and principle.

Guideline to which the diagnosis of concept and principle using difficulty refers, according to [4], is elaborated as follows: (1) diagnosis of concept using difficulty includes marking/defining concept, identifying example and non-example, using multiple representation, identifying, comparing, and confirming concept; (2) diagnosis of Principle using difficulty includes: identifying with when and why a principle is needed, identifying and using principle correctly, identifying the correct and incorrect principle, generalizing new principle and modifying a principle, appreciating the role of principles in mathematics.

Diagnostic tests are constructed to reveal the students' weakness in certain part of a subject rather than to assess the achievement in the subject, and even the errors made in achievement tests can indicate the students' difficulty. Trigonometry conceptual understanding can be developed by exploring multiple representations, function property, and its connection to the knowledge the students have learnt. However, when students have weak or even incomplete understanding, they will find difficulty in learning trigonometry function. The characteristics of students' learning difficulty, according to [10], are closely related to the objective of learning, related to content and learning process. Considering the elaboration above, the difficulty of learning trigonometry function can be identified from students' error in working on conceptual understanding problems including: multiple representations, function property, and its connection to prerequisite concepts. The result of research shows that a variety of sufficiently complex trigonometry learning difficulties in students. The difficulty in understanding the prerequisite material concept is found in determining function and non-function. Students have their own language in defining function. Students find difficultly the reason of why a chart or an equation is considered as function or non function. Sometimes, students understand process perspective, but they often determine the value constituting input (domain) and the one constituting the range incorrectly.

Some studies [3], [11], and [20] found that students define difficultly sine and cosines as the function with real number domain. It is likely, according to [18], due to pi concept in trigonometry contexts conceived differently from that in real number context. Trigonometry function domain is the angle size large set commonly expressed in degree or radian. The understanding on radiant concept, according to [1], is very important to successful trigonometry function learning. Radian concept can be used to connect the representation of function numerically and graphically, creating a definition of function based on unit circle [2]. Radian can also be used to show that trigonometry function domain is real number. Some studies [5], [14], [18], [1], and [19] revealed that students, college students, and even teachers find difficulty in understanding radian concept. Tuna [19] revealed that prospect teachers do not conceive the concept of degree correctly. Nevertheless, the findings of other studies [19] and [1] showed that students are accustomed to using more degree concept than radian concept. Considering the result of studies on students' difficulty in learning trigonometry, it can be concluded that the students' types of difficulty are as follows: (1) difficulty of understanding prerequisite concept includes: determining and distinguishing function from non-function, determining domain and range, identifying that trigonometry function domain involves real number, radian concept, degree concept, and pi concept; (2) the difficulty of representing function: representing function graphically, identifying the relationship between

graphic and function equation, representing numerically and verbally; (3) difficulty of connecting the relations between function representations; and (4) difficulty of understanding the property of periodic function: amplitude, period, domain, and range.

Overall, diagnostic test was employed in this study to find out the students' difficulty in using concept and principle. The difficulty of using concept intended includes: prerequisite concept, concept representation, and property of trigonometry function graphic (chart). Meanwhile, the difficulty of using principle intended is the one in coordinating the combination of several concepts and several facts created through operation or relation.

3. Remedial Teaching to Overcome Trigonometry Learning Difficulties

Having studied the result of diagnostic test on trigonometry learning difficulty, remedial teaching is conducted as an alternative solution to students' difficulty using Casio Classwiz fx-991EX calculator. Data of diagnostic test result was analyzed, supported with the result of interview and its transcript. The subjects of interview were selected representing any type of error in the 1st diagnostic test. The result of Trigonometry Diagnostic Test on 96 students in SMK (Vocational High School) X representing the class with high, medium, and low abilities would be classified into four types of problem answer: (1) correct or nearly correct, (2) wrong-concept, procedure, or principle answer, (3) not answering, and (4) other type of answer. This classification accommodates the technique used by [6].

The result of test for item number 1 about size concept in degree system shows that 15 out of 96 students (15.6%) identify degree as one of size measurement units, recognize degree concept as the result of final line rotation, and know that 1 degree is 1 part of 360 parts in a whole circle. Such understanding is not completely correct. Students know degree concept in circle with any size of radian. Students do not understand degree concept in a unit circle. Meanwhile, 55 out of 96 students (57.2%) conceive angle concept as a result of angle measurement. In line with [17] and [19], it can be seen that students capture the concept of angle unclearly, thereby conceiving it as the result of angle measurement. This, according to Thomson, results not only from wrong concept, but also from the presentation of angle in textbook presenting the figure of angle by pointing to the size rather than to its creation process. Such books, according to Thompson, still follow the old model presentation. The modern calculus books have presented angle as the result of ration in a unit circle. It can also be demonstrated during the learning, to make the students conceiving the concept of angle as its creation process rather than as its size. In addition, 15 out of 96 students (15.6%) know that degree is one of angle measurement units but they do not give explanation. Meanwhile, 11 out of 96 students (11.5 %) experience concept error. Generally, it can be said all test participants do not understand degree concept clearly. The result of research identifies the students' difficulty in understanding angle as rotation process rather than the result of angle measurement. The difficulty of understanding angle concept in degree will result in students' difficulty in understanding angle concept in radian and trigonometry function concept.

The discussion of item number 2 about concept of angle in radian system shows that no student understands concept of angle in radian. Concept error is shown by 32.2% of students explaining that 1 radian is the measurement result of 1 radian. About 18.75% of students do not respond to the question. Meanwhile, 48.9% of students experience concept error considering that 1 radian is 1 whole circle in a circle or angle size of 360^o and some other consider it as a half of circle or pi. It is in line with a study [18] finding that only 8% of prospect teachers can define radian concept correctly. It indicates that students experiencing not-meaningful learning process thereby forgetting the concept learnt easily. As suggested by [6], radian concept is explained briefly and emphasized more on the process of converting radian angle size into degree, minute, and second. The conversion procedure is consistent with the example given in the book. Thus, students only follow the procedure without

understanding the meaning. Overall, it can be concluded that all test participants find difficulty in using and understanding radian concept. Considering the result of interview, it can be found that students rarely use arch, compass, and calculator. The problems given are dominated with special angles needing memory only to use it. It means that the learning emphasizes on cognitive aspect only and ignores psychomotor one.

The result of diagnostic tests in item number 3a still concerning the relation between degree system and radian shows that 11 students (11.5%) give correct answer in radian. Procedure error is shown by 15 students (15.6%). About 62 (64.6%) of 96 students do not respond to the problem. Meanwhile, 8 (8.3%) of 96 students find difficulty in determining principle or procedure that will be used to solve problem. Most students try to make illustration with actual size and by measuring the object directly, but the result is incorrect. It is in line with Thompson (2013: 7) finding that only 14% of prospect teachers can solve this type of problem correctly. Meanwhile in item number 3b, only 7 (7.2%) students answer correctly in degree. About 68 (70.8%) of 98 students do not respond to the problem. About 21 of 96 (22%) students find difficulty in determining principle or procedure to be used to solve problem, and some students try to make illustration with actual size and by measuring the object directly, but the result is incorrect. Considering the result of test and interview, it can be found that the students not only find the difficulty of understanding radian and degree concept, but also cannot convert it skillfully. The result of research indicates that the presentation of degree conversion into radian and vice versa illustratively and conceptually. Therefore, remedial teaching reveals again how to make the students understand the conversion of angle size in radian and degree systems. The students are given activity sheet and guidance using Casio Classwiz fx-991EX calculator.

Radian measure provides a different way of measuring angles from the use of degrees, by measuring along a circle. An angle has measure of 1 radian if it cuts an arc of a circle that is one radius long. As there are 2π radius measures in the circumference of a circle, the measure of a full rotation a circle is 2π radians. The same angle is 360^0 in degrees (or sexagesimal measure). In this case, to understand the equality of values in the two systems, the students can work on the exercise leading to cognitive reasoning using calculator. By setting up the calculator menu in radian, and then inputting value in radian, followed with set up degrees, the equal value in degree system is obtained, and vice versa. Systematic worksheet will of course help students to construct their own understanding.

The result of test as illustrated in Figure 3.1 about pi concept in item number 4 shows that no student understands pi concept clearly. About 75 of 96 (78%) students state that to find breadth or circumference of a circle, use $\pi = 3.14 = 22/7$, while to work on trigonometry, use $\pi = 180^0$. The students do not know that pi is approached by 3.14… and $\pi \neq 3.14 \neq 22/7$. Even no student considers that $\pi = 3.14 = 22/7 = 180^0$. About 9 of 96 (9.4%) students do not respond to the question.

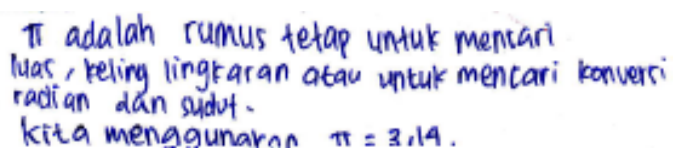

Figure 3.1 Answer to Item number 4 (Student S2)

Meanwhile, 12 of 96 (12.5%) students understand concept error: (1) considering that pi is the number of angles in triangle, and (2) pi is degree size unit. This condition can be overcome by recalling past labs to know the concept of pi. Then by comparing the conversion table data using two calculators, students are asked to find the equivalent of radians by

degrees. In the end students understand that a radian is a little more than 57^0, and that a full circle of 360^0 has a measure of 2π radians.

The result of test on item number 5 shows that 67 of 96 (70%) students can solve congruence problem in item number 5a. About 11 of 96 (11.5%) students experience procedure error in calculating the final result. About 18 of 98 (18.8%) students do not respond to the question. Meanwhile, in item number 5b, 20 of 96 (21%) students can give reason and connect it to tangent function. About 46 of 96 (47.9%) students give no reason and do not connect it to tangent function. Thirty of 96 (31.25%) students do not respond to the question.

The result of test on item number 6 about determining sine, cosines and tangent values shows that only 3 of 96 (3%) students can solve the estimated value of $\sin 50^0$. About 8 of 96 (8%) students experience procedure error in calculating the final result. Students try to solve problem using $\sin\alpha = \sin(A \pm B)$ formula. About 72 of 96 (75%) students do not respond to the question. Meanwhile, 11 of 96 (13.5%) students try to make 50^0 angle in a circle, then make equilateral triangle, and apply the side comparison formula for sine function.

The result of test on item number 7 about how to determine the length of one sides in right triangle, when tangent value of one of its angles is known, shows that 23 of 96 (24%) students can solve the problem correctly. About 10 of 96 (10.4%) students experience procedure error in calculating the final result. About 47 of 96 (49%) students do not respond to the question. Meanwhile, 16 of 96 (16.7%) students do not pay attention to text information and figure. Students work on the problem directly by seeing the numerator value as y, and denominator value as x.

The result of test on item number 8, determining value of $\sin\frac{1}{2}, \cos 130^0$, and $\tan\frac{7}{6}\pi$, shows that no students (0%) answer the item number 8a correctly. About 10 of 96 (10.4%) students experience procedure error in calculating final result. The fact can be seen from figure 3.2 below.

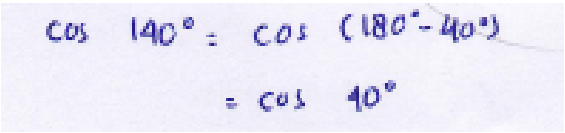

Figure 3.2 Answer to Item Number 8a (Student R2)

About 61 of 96 (63.5%) students do not respond to the question. Meanwhile, 25 of 96 (26%) students do not know the fact of radian notation, thereby not understanding the intention of question. The students think that the item is typed incorrectly, and reveal that in their knowledge there is no problem with sin ½, there is only a problem with 30 degree sin. The fact can be seen from Figure 3.3 below.

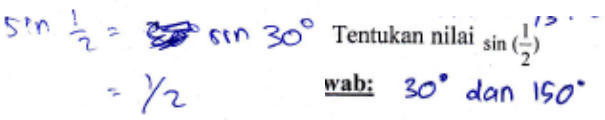

Figure 3.3 Answer to Item Number 8a from (Students T2 and S2)

Meanwhile the item number 8b shows that no student gives correct answer. However, 15 of 96 (15.6%) students use ratio system thereby indicating that value $\cos 130^0 = -\cos 50^0$. Furthermore, students predict difficultly the cosines value of non-special angles. About 9 of 96 (9.3%) students experience procedure error in determining $\cos 130^0 = -\cos 50^0$. About 72 of 96 (75) students do not respond to the question. Considering the result of interview with student S1, it can be seen that student's assumption about non-special angles cannot be determined by trigonometry function value. He/she says that this type of question has never been learnt. It means that students are not accustomed to and even do not know that trigonometry function is a continuous function, the line containing points as the solution to the function. Some students assume that only special angles have trigonometric function value, while other angles have no trigonometric function value.

The result of test on item number 9 about contextual problem determining the height of kite shows that no students (0%) answers correctly. But, 25 of 96 (26%) students use ratio system, thereby indicating that the height of kite = $30 \times \sin 50^0$. It is illustrated in Figure 3.4.

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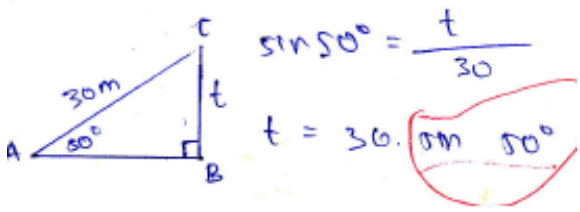

Figure 3.4 Answer to Item number 9 (Student S2)

Furthermore, students find difficulty in predicting sine value of non-special angle. About 6 of 96 (6.25%) students experience procedure error in determining the final result. Fifty eight of 96 (60.4%) students do not respond to the question. Meanwhile, 7 of 96 (7.3%) students try to predict value of $\sin 50^0$ but they make some error. The result of test on item number 10 about concept of sine function shows that no student defines sine with definitions of function or ratio or others. About 83 of 96 (97%) students do not respond to the question. Meanwhile, 3 students define the function by mentioning the property of function: (1) function starting from 0 degree (domain), (2) function having such graphic (chart) (drawing sine graphic), (3) function having hill and valley. Considering the result of interview, it can be seen that students find difficulty in distinguishing function from non-function graphically and numerically. This condition can be overcome by associating between function representations that include verbal, symbolic, numerical, and graphical. To observe graphical representation, students can install applications from playstore i.e. Casio edu +. By observing the graph of the trigonometric function and then recognizing its attributes numerically using the classwiz casio calculator, students are expected to be able to recognize the function.

Moreover, the casio classwiz fx-991EX calculator with assisted worksheets that support cognitive exploration not only can help students overcome the learning difficulties of trigonometry, but also help develop students' reasoning abilities.

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